Hawking radiation

- Renaud Parentani*, Laboratoire de Physique Théorique d'Orsay, Univ. Paris-Sud 11, France
- Philippe Spindel, Université de Mons, Belgium

Hawking radiation is the thermal radiation predicted to be spontaneously emitted by black holes. It arises from the steady conversion of quantum vacuum fluctuations into pairs of particles, one of which escaping at infinity while the other is trapped inside the black hole horizon. It is named after the physicist Stephen Hawking who derived its existence in 1974. This radiation reduces the mass of black holes and is therefore also known as *black hole evaporation*.

Contents

Black hole formation

There are two kinds of black holes in the Universe: those of stellar origin of a few solar masses, and those found in globular clusters or in galactic nuclei. The second are much more massive; their masses vary between a few hundred and a billion solar masses. The first type are better known, and we briefly review how they form. When a sufficiently massive star has burned its nuclear material, its internal pressure is no longer able to resist its own gravitational attraction. As a result, the star implodes. The outer layers bounce off the inner ones and a large fraction of the star's matter is ejected at a speed on the order of a few percent of the speed of light c .The star undergoes a supernova. It will then contract and, if the residual material is not too massive, a new equilibrium state will be reached: a *neutron star*. But if the mass is greater than a few solar masses, the pressure will not be able to counterbalance its weight. It will thus ineluctably keep collapsing and form a black hole.

Black hole horizon properties

From a more geometric point of view, in the theory of general relativity, a black hole is a region of spacetime characterized by a boundary called the *horizon* that separates the outer region -– from which light rays can escape and reach far distant observers -– from the trapped region -– from which neither matter nor light can possibly escape (see Figure 1 and [1]). The simplest example is a stationary, non-rotating black hole. In this case, at each instant, the horizon is the surface of a sphere. Its area is equal to $4\pi r_S^2$,where the *Schwarzschild radius* r_S is related to the black hole mass M by

$$r_S = \frac{2GM}{c^2}, \qquad (1)$$

where G is Newton's gravitational constant $= 6,674\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$ .For a one solar mass black hole, r_S is approximately equal to $3\,\mathrm{km}$,i.e. much smaller than the present radius of the Sun which is of the order of $7 \times 10^5\,\mathrm{km}$.

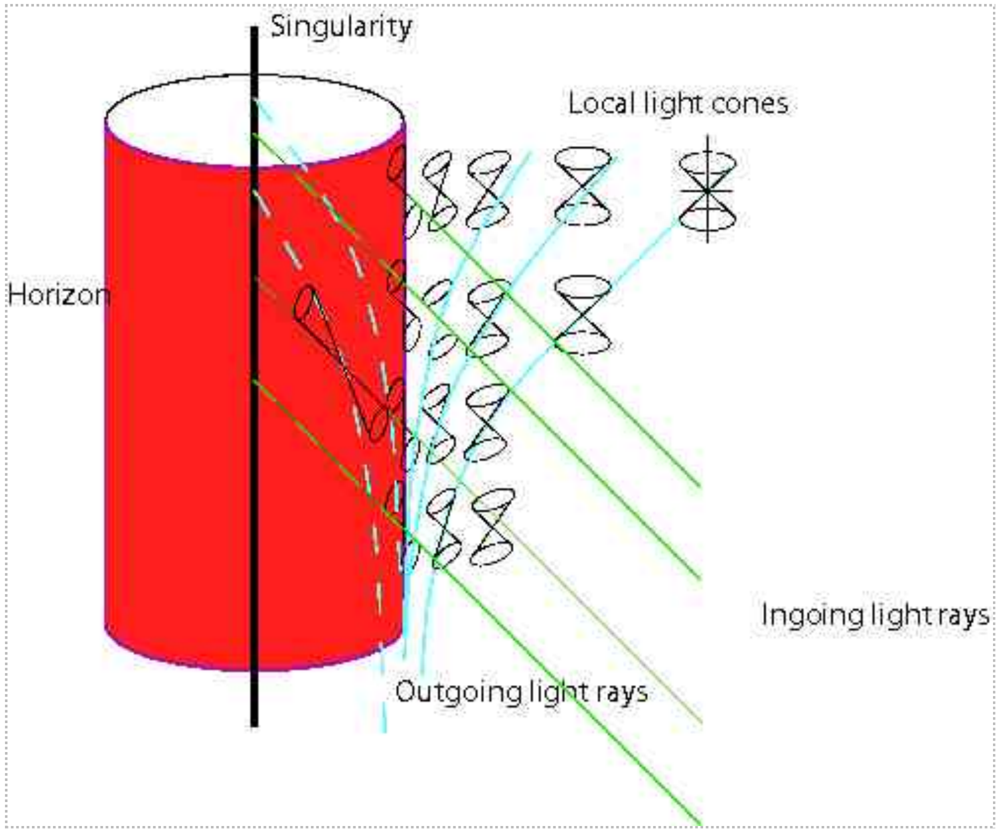

Figure 1: We have represented the spacetime geometry of a spherical black hole. The time coordinate increases vertically, and one of the three spatial dimensions is not shown. Thus, at a given time, i.e. on a horizontal plane, the horizon appears here as a circumference of length $2\pi r_S$,and not as the surface of a sphere. This geometry is static, and in particular the horizon keeps the same surface area at all times. One clearly sees that the horizon is a cylinder that separates the trajectories of outgoing light rays in two classes: those represented in blue that escape the hole, from those represented in dotted lines which are trapped inside. None of them cross the horizon. Instead radially infalling light rays, here represented by straight green lines, cross the horizon without hindrance.

When considered at all times, the horizon forms a three dimensional cylinder (embedded in the four dimensional spacetime) whose base is the spherical surface we just discussed, and whose third dimension is engendered by straight lines. What is peculiar about these lines is that they are part of some future light cones, as can be seen in Figure 1. (In relativistic jargon, they are called "null" lines because the *spacetime* distance between any two points of one of them is exactly zero.) More precisely, these lines are the outermost generators of the forward light cones whose vertices are situated on the horizon itself. Because of the curvature of the black hole spacetime, these lines stay on the surface of fixed area $4\pi r_S^2$ instead of spreading from each other as it is the case in flat spacetime. In this sense the black hole horizon is static and eternal.

More physically, the above results imply that no light rays emitted from these vertices could possibly propagate outwards, i.e. with increasing values of the radial coordinate r . The best they can do is to slide at fixed $r = r_S$, along the horizon.

Redshift and structure of outgoing light rays

The classical theory of gravitation also predicts that Ω_0 , the frequency of some light pulse emitted by infalling matter, is redshifted in its outwards journey. More precisely, when the matter is about to cross the horizon, the frequency received by observers very far from the black hole and at rest with respect to it decreases following an exponential decay law:

$$\omega(t) \sim \Omega_0 e^{-(t-t_0)/\tau_\kappa}. \tag{2}$$

It is thus redshifted to zero with a characteristic lifetime given by τ_κ . This characteristic time depends only on the mass of the black hole**1**, and it is related to the Schwarzschild radius of the horizon by

$$\tau_\kappa = \frac{2 r_S}{c}. \tag{3}$$

We see that it is given by the time it takes light to travel a distance equal to $2r_S$ .For a one solar mass black hole, one finds that $\tau_\kappa \approx 2 \times 10^{-5} s$ .Even though the decay law of Eq. (#Cdecay) continues indefinitely, it implies that after one second no light is received far away from the hole since $e^{-5\times 10^4} \approx 10^{-21715}$ is an extremely small number, indistinguishable from zero in practical terms. Since this redshift also applies to the light emitted by the collapsing star, it means that after one second the collapsing object is effectively black. At this point it should be also mentioned that the trajectories of the outgoing light rays that experience the redshift of Eq. (#Cdecay) separate from the horizon exponentially fast. Indeed, near the horizon, they are given by

$$r(t) - r_S \sim (r_0 - r_S)\, e^{(t-t_0)/\tau_\kappa} \tag{4}$$

This is the inverse law of Eq. (#Cdecay), and it is governed by the same characteristic time τ_κ . In addition Eq. (#Cdecay2) also applies to the *outgoing*, i.e. outermost travelling, light rays that are trapped inside. In that case, $r(t) - r_S$ is negative since $r(t) < r_S$ .The horizon at $r = r_S$ thus separates these two families of outgoing rays, as illustrated in Figure 1.

If Nature were not fundamentally quantum mechanical, this would be the end of the story: a collapsing star would stop radiating at the end of its collapse in a fraction of a second and would thereafter never radiate. However the black hole would still gravitationally attract matter and light. As a result infalling matter will cross the horizon and enter into the trapped region, thereby justifying why such an object has been called a black hole. It should be noticed that through this accretion, the mass of the black hole will necessarily increase but cannot decrease. This resembles the second law of thermodynamics which stipulates that the entropy of a system can never decrease.[2] For more details, we refer the reader to the contribution of J. Bekenstein which is devoted to this interesting correspondence of deep consequence.

Quantum mechanics and Hawking radiation

When taking into account the quantum properties of light (which were so far ignored), to the great surprise of his colleagues and himself, Stephen Hawking discovered in 1974 that newly formed black holes are not black.[3] Indeed he found that they spontaneously emit a steady thermal flux of radiation at a temperature given by

$$k_B T_{\text{Hawking}} = \frac{\hbar}{2\pi\tau_\kappa}, \tag{5}$$

where k_B is the Boltzmann constant $1.4 \times 10^{-23}\text{J/K}$, and $\hbar = 1.06 \times 10^{-34}\text{J s}$ is the Planck constant.

Using the quantum mechanical relation $E = \hbar\omega$ between energy and frequency, and the relation between energy and temperature $E = k_B T$,one sees that the typical frequency ω associated with this thermal radiation is given by $1/\tau_\kappa$, up to a constant factor of 2π .In order words the typical frequency of thermal flux is fixed by the decay rate of Eq. (#Cdecay).

The above equations fix the numeral value of the temperature to

$$T_{\mathrm{Hawking}} = \frac{6.2 \times 10^{-8}\mathrm{K}}{m_\odot}, \quad (6)$$

where $m_\odot = M_{BH}/M_\odot$ denotes the black hole mass expressed in the units of the solar mass $M_\odot = 2 \times 10^{33}\mathrm{gr}$ For black holes of a few solar masses it is thus extremely small, much smaller than that of the cosmic microwave background which is of the order of $3\,\mathrm{K}$. As a result, these black holes will absorb more radiation than they will emit, making their mass increase. It is only in a very very far future, when the temperature of the microwave radiation will be reduced -- due to the expansion of the Universe -- below their temperature, that they will start losing mass through the emission of Hawking radiation. Even though their temperature will progressively increase, as it scales as the inverse of their mass, the evaporation will be extremely slow. Typically it will last a time on the order of

$$\tau_{BH} \approx m_\odot^3\, 10^{74}\, s. \quad (7)$$

For a solar mass black hole it is much greater than the age of the Universe: fourteen billion years.

It should be noticed that the Hawking effect does not concern only light, but all kinds of elementary particles. Indeed, all particles in Nature are described by quantum fields which behave essentially like the quantum radiation field describing light. However, for a solar mass black hole, the temperature is so low that the thermal energy $E_T = k_B T_{\mathrm{Hawking}}$ is much smaller than the rest energy $E_m = mc^2$ of massive particles, such as the electron. As a result, solar mass black holes would effectively emit only massless particles. This would remain true until their residual mass has sufficiently decreased so that the thermal energy E_T would have sufficiently increased so as to have reached the rest energy E_m of the lightest massive particles. For simplicity in the forthcoming discussion, we shall only consider light quanta.

The origin of Hawking radiation

Let us now explain the mechanism that is responsible for this thermal flux. It is to be found in the redshifting effect of Eq. (2) and the associated tearing apart of the light rays across the horizon of Eq. (Figure). The exponential redshifting applies individually and universally to all light waves irrespectively of their initial frequency Ω_0 . A closer examination shows that every wave, in addition of being redshifted, is slightly amplified by this redshift. Moreover, it can be also shown that this amplification is necessarily accompanied by a correspondingly small production of a partner wave of opposite frequency. In classical terms, these two effects have no significant consequences because they are weighted by the amplitude of the partner wave (the coefficient β_ω in Eq. (8) below) which is in general extremely small. On the contrary, in quantum mechanical terms this small amplification accompanied by the production of a partner wave is of uppermost importance as it is directly responsible for the Hawking effect. In quantum terms indeed, in the vacuum, the state of minimal energy, *nothing* would have happened without this amplification, i.e. black holes would have remained black.

Given the importance of this amplification, let us describe it with more precision. When considering the propagation of light in the static spacetime obtained after the collapse, one finds that outgoing wave packets initially localized very near the horizon split into two waves: one with positive frequency that escapes and a partner wave $\phi_{-\omega}$ of negative frequency which is trapped inside the horizon:[2]

$$\phi_\omega^{\mathrm{initial}} = \alpha_\omega \phi_\omega^{\mathrm{escaping}} + \beta_\omega \phi_{-\omega}^{\mathrm{trapped}}. \quad (8)$$

There is a conservation law associated with the splitting that takes the form

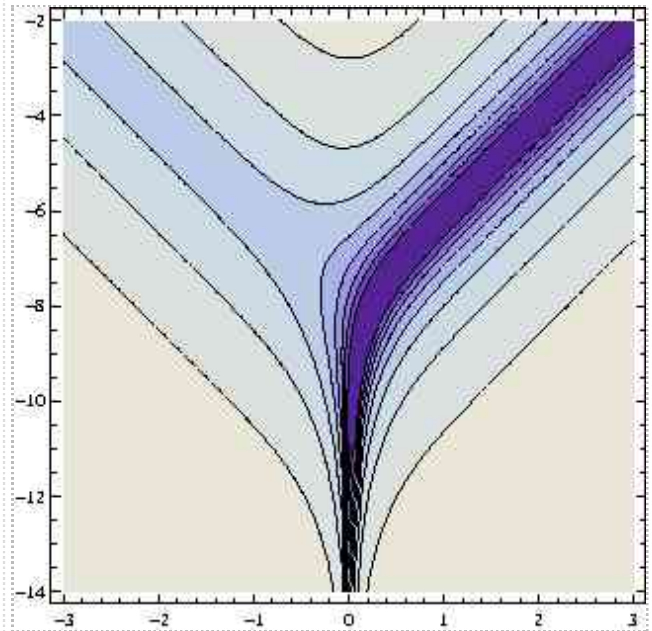

Figure 2: The spacetime pattern of the energy flux density associated with a typical wave $\phi_\omega^{\rm initial}$ of Eq. (8), taken from [4]. As in Figure 1, the time coordinate increases vertically. The horizontal coordinate is the radial coordinate r , and the two angular coordinates are not shown. Hence the three dimensional horizon of radius r_S is represented by a vertical line which is here in the middle of the figure. At a later time, one clearly sees the trajectory towards the upper right corner followed by the escaping wave packet and that of trapped partner wave propagating inward. The lighter colour of the latter expresses the fact that its amplitude β_ω in Eq. (8) is much smaller than α_ω , the amplitude of the escaping wave. One also sees that these trajectories are basically the same as those of outgoing light rays represented in Figure 1. At an early time, before they split, there is a unique initial wave packet. When reconsidered in quantum mechanical terms, this splitting of waves describes the steady conversion of vacuum fluctuations into pairs of particles.

$$|\alpha_\omega|^2 = 1 + |\beta_\omega|^2. \qquad (9)$$

This law relates α_ω , the amplification factor of the escaping wave, to β_ω , the amplitude of the partner wave which is trapped. Hawking showed that these coefficients obey

$$\left|\frac{\beta_\omega}{\alpha_\omega}\right|^2 = e^{-2\pi\omega\tau_\kappa} = e^{-\hbar\omega/k_B T_{\rm Hawking}}. \qquad (10)$$

Recalling that the Boltzmann law of thermal equilibrium takes the form $e^{-E/k_B T}$, and re-using the relation $E = \hbar\omega$, one can read from Eq. (10) the Hawking temperature of Eq. (5).

It now remains to understand what happens to the vacuum state when the amplification factor $|\alpha_\omega|^2 > 1$, i.e. when β_ω does not vanish. To this end, one must recall that the field describing quantum light does not strictly vanish in the vacuum. In fact, the field steadily fluctuates around a vanishing mean value.**3** In usual circumstances, these vacuum fluctuations remain unchanged, thereby expressing the stability of the vacuum state. However, when they are excited by some external agent, there can be a quantum transition which is accompanied by the emission of a photon, i.e. a excitation of the quantum field of light. For instance, when the light field is coupled to an excited atom, this causes the spontaneous decay of the atom and the emission of a photon. Similarly here, the redshifting effect of Eq. (2) excites some vacuum fluctuations and this leads to the steady production of photons. As for the spontaneous decay of atoms, the moments when the production occurs are randomly distributed. Indeed quantum mechanics only fixes the mean rate of their occurrence. A detailed calculation shows that this production rate is constant and fixed by $|\beta_\omega|^2$, the squared norm of the partner wave coefficient that appears in Eqs. (8,9,10). For more details on this correspondence we refer to the review article [5].

Two important differences between atomic transitions and black hole radiation should be underlined. The first difference between coupling light to atoms and to the gravitational black hole field is that the latter necessarily leads to the production of *pairs* of photons. It can also be shown that in each pair, one photon escapes to spatial infinity and carries a positive energy $\hbar\omega$, whereas its partner carries a negative energy $-\hbar\omega$, and remains trapped inside the horizon. Moreover, in each pair, the two photons are "entangled", i.e. correlated with each other. Their entangled character can be revealed by studying non-local correlations across the black hole horizon. Doing so one obtains a spacetime pattern which is similar to that associated with the splitting of Eq. (8), see Figure 2. A second difference is that these pairs are steadily produced, one after the other, at the expense of the black hole mass. The black hole effectively behaves as an extremely excited atom that would have stored a huge amount of energy and would release it extremely progressively, as can be seen from Eq. (7) which gives the enormous lifetime of black holes. Taken together the escaping members of these pairs form a thermal flux at the Hawking temperature.

Observing Hawking radiation

The verification of Hawking's prediction by astrophysical observations is most probably impossible, due to the low temperature of massive black holes, see Eq. (6), and the (apparent) non-existence of black holes of much smaller mass. So far indeed no small mass black holes have been observed, and moreover there are good astrophysical reasons to believe that no such black hole should exist in our neighborhood.

Fortunately, as pointed out by William Unruh in 1981[6], there exist physical systems which display a profound analogy with the Hawking radiation and which are susceptible to be observed in the lab, see [11]. One of these consists of sound waves traveling in an accelerating fluid that flows across a bottleneck where it reaches supersonic velocity. Sound waves propagating against the flow may row up the stream where the fluid velocity is subsonic, but they will be dragged down stream where the velocity is supersonic. In addition, the sound waves will stay at the same place where the fluid velocity is equal and opposite to the sound speed. All these properties are in perfect analogy with those of outgoing light rays propagating near the black hole horizon: In the present case, there is a sonic horizon that separates the fluid into two regions by a one way boundary, just like the black hole horizon does for light. In fact, Figure 1 also describes the "sound rays", i.e. the trajectories of sound wave packets: the tearing apart of the sound rays across the sonic horizon follows Eq. (Figure), and the sound frequencies decrease following Eq. (2). In these equations, the characteristic time τ_K is no longer given by Eq. (3) but by the gradient (the steepness) of the velocity flow at the sonic horizon where it crosses the speed of sound.

This analogy becomes even more precise when comparing the equation governing sound propagation in this flow and that governing light propagation near a black hole horizon. Indeed, when considering long wavelengths, i.e. ignoring the molecular properties of the fluid, the two wave equations are identical. On this basis, Unruh predicted that a flow possessing a sonic horizon should spontaneously produce a thermal flux of phonons, the quanta of the phonon field, for the same reasons that a black hole horizon should emit photons, the quanta of the light field. The analogy works so well that the temperature of this analogous radiation is also given by Eq. (5). On should simply use the value of τ_κ that governs the sound frequencies decrease of Eq. (2). Therefore if one were able to detect the spontaneous emission of phonons by an analogue black hole, one would experimentally validate Hawking's prediction, albeit in an analogue situation.

So far however, the smallness of the temperature of emitted phonons has prevented the observation of the analogue radiation. To ease its detection, one should envisage, and possibly realize in the near future, situations where the analogous Hawking radiation will be amplified. For instance, when a supersonic fluid slows down to subsonic velocities, another sonic horizon is created. This new horizon is analogue to that of a white hole, the time reverse of a black hole. It can be shown that the pair of horizons acts as a resonating cavity which can lead to an important amplification of Hawking radiation.[7]

Let us mention that several experiments have been conducted in 2010. The most conclusive one is that performed in Vancouver.[8] They analysed the scattering of surface waves propagating in a water tank against a flow towards an analogue white hole horizon. This situation somehow corresponds to the time reverse of that represented in Figures 1 and 2. By careful measurements of the amplitudes of the various waves, they were able to measure the ratio of the amplitudes α_ω and β_ω appearing in Eq. (8). Quite remarkably their results are in accord with the theoretical law of Eq. (10), with the characteristic time τ_κ given (within error bars) by the gradient of the velocity flow at the horizon. The whole experience dealt with classical waves produced by a generator and not with quantum mechanical vacuum fluctuations. Yet what they observed, the conversion near the horizon of incident waves into two outgoing waves of opposite frequency, see Eq. (8), is very important because it is this the mechanism that causes the Hawking radiation at the quantum level. In other words, they have observed the induced (stimulated) Hawking radiation, and not the spontaneous one associated with vacuum fluctuations. The fact that Eqs. (8),(9),(10) govern both classical mechanics (stimulated effects) and quantum mechanics (vacuum effects) follows from the linear character of wave equations.

Quantum gravity and the role of very short distance effects

To conclude, we wish to point out that the discovery of Hawking has raised deep questions, and triggered many developments in theoretical physics. For instance, much could also be said about the developments concerning Hawking radiation and black hole entropy within string theory. For these interesting subjects, we refer for example to the book [9].

Following the preceding section, we point out that the analogy between sound and light, fluid mechanics and gravity, sheds some light on a puzzling conceptual problem that arises when deriving the Hawking radiation. The question concerns the role of extremely high energies at the origin of this radiation. Indeed, when read the other way around, the exponential redshift of Eq. (2) implies that the Hawking photons emitted at a late time possessed in the past exponentially high frequencies Ω_0 near the black hole horizon. This means that, for a one solar mass black hole, a typical photon arriving at spatial infinity with an energy of the order $10^{-11}\mathrm{eV}$ in fact issued from vacuum fluctuations of energy much greater than the Planck energy: $10^{28}\mathrm{eV}$ and concentrated in a domain smaller than the Planck length: $10^{-33}\mathrm{cm}$ .Here we touch at the *terra incognita* of the present day physics. To describe what happens at that scale requires a quantum theory of gravity, something many have tried to build without success as yet. This exponential growth of the energies involved has, sometimes, been advocated as an indication that Hawking radiation may not exist! However such a scenario raises in turn numerous difficulties. Indeed, Hawking radiation is related to the fact that black holes posses a (finite) entropy and thus a (non zero) temperature: the Hawking temperature. Hence the absence of the Hawking radiation would lead to violations of thermodynamical laws.

When considering the analogue question phrased in the framework of sonic horizons, the situation is quite different because we known what happens at very short distance: molecular physics furnishes a microscopic scale below which the sound propagation is altered by dispersive effects. This implies that the early propagation of sound pulses differs from that represented in Figures 1 and 2 in the vicinity of the horizon when approaching this microscopic scale.[8][10] In spite of this, it turns out that short distance dispersion does not alter significantly the outcome, i.e. the production of a thermal flux with a temperature fixed by the characteristic decay rate $1/\tau_K$. From this we learn that the Hawking effect is a robust prediction of quantum fields propagating in the vicinity of a horizon, rather insensitive to the peculiar properties of the theory at very short distances. In this sense, studying analogue black holes has reinforced the unexpected prediction that black holes radiate.

We are grateful to J. Bekenstein and T. Jacobson for useful remarks on an early version of this work.

Footnotes

1. When the black hole is rotating, the characteristic time τ_K also depends on its angular momentum.
2. Unlike $\omega(t)$of Eq. (2), the frequency ωin Eq. (8) is constant in time: static observers far away from the black hole would find it constant. In fact, the exponential relation of Eq. (2) arises from the fact that the initial frequency Ω_0 was defined in the frame of the infalling matter which radically differs from the inertial frame at infinity when the matter is approaching the horizon.
3. This is a direct consequence of the Heisenberg uncertainty relations. Indeed if a field amplitude were strictly vanishing forever, this would imply that we would exactly know both its value (zero) and its time variation (also zero). Such knowledge is not possible in the framework of a quantum theory and, as a result, dynamical variables, such as field amplitude, are bound to fluctuate steadily.

References

1. B. Carter B., *Black Hole equilibrium States*, in *Black Holes. Les Astres Occlus.*, Proceedings of Les Houches Summer School on Theoretical Physics, edited by C. DeWitt and B. S. DeWitt, Gordon and Breach (1972).
2. J. D. Bekenstein, *Black holes and entropy*, Phys. Rev. D **7**:2333-2346 (1973), and J.M. Bardeen, B. Carter, B. and S.W. Hawking, Commun. Math. Phys. **31**, 161–170 (1973).
3. S.W. Hawking, *Particle creation by black holes* Commun. Math. Phys. **43**, 199–220 (1975), and *Black hole explosions?* Nature **248** (5443): 3031 (1974) .
4. R. Parentani, *From vacuum fluctuations across an event horizon to long distance correlations*, Phys. Rev. D **82** 025008 (2010).
5. R. Brout, S. Massar, R. Parentani and Ph. Spindel, *A Primer for black hole quantum physics*, Phys. Rept. **260** 329 (1995). [arXiv:gr-qc 0710.4345].
6. W. G. Unruh, *Experimental black hole evaporation*, Phys. Rev. Lett. **46**: 1351-1353 (1981), and *Sonic analog of black holes and the effect of high frequencies on black hole evaporation*, Phys. Rev. D **51**: 2827-2838, (1995).
7. S. Corley and T. Jacobson, *Black hole lasers*, Phys. Rev. D **59** 124011 (1999), and A. Coutant and R. Parentani, *Black hole lasers, a mode analysis*, Phys. Rev. D **81** 084042 (2010).
8. S. Weinfurtner, E. W. Tedford, M. C. J. Penrice, W. G. Unruh and G. A. Lawrence, *Measurement of stimulated Hawking emission in an analogue system*, Phys. Rev. Lett. **106** 021302 (2011). [arXiv:1008.1911 [gr-qc]].
9. E. Kiritsis, *String Theory in a Nutshell*, Princeton University Press (2007).
10. R. Brout, S. Massar, R. Parentani, and Ph. Spindel, *Hawking radiation without transPlanckian frequencies*, Phys. Rev. D **52**: 4559-4568, (1995). [arXiv: hep-th/9506121]
11. T. Jacobson and R. Parentani, *An echo of black holes*, Scientific American, **17**, 12-19 (2007).

See Also

Bekenstein bound, Bekenstein-Hawking entropy, Black holes, Entropy

Sponsored by: Riccardo Guida, Institut de Physique Théorique; CEA, IPhT; CNRS; Gif-sur-Yvette, France

Sponsored by: Eugene M. Izhikevich, Editor-in-Chief of Scholarpedia, the peer-reviewed open-access encyclopedia

Sponsored by: Patrick Peter, Gravitation Relativiste et Cosmologie, Institut d'Astrophysique de Paris, FR

Reviewed by (http://www.scholarpedia.org/w/index.php?title=Hawking_radiation&oldid=91348) : Pietro Menotti, Pisa University, Italy

Reviewed by (http://www.scholarpedia.org/w/index.php?title=Hawking_radiation&oldid=0) : Riccardo Guida, Institut de Physique Théorique; CEA, IPhT; CNRS; Gif-sur-Yvette, France

Category: Space-time and gravitation