Maximum-entropy closure for unsteady fluid flows

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Since its beginning, statistical turbulence modelling has been inspired by the simplicity and elegance of statistical physics description of an ideal gas ('molecular chaos'). Prandtl's (1925) mixing-layer theory, for instance, adopts main elements of gas kinetics in estimating Boussinesq's (1877) eddy viscosity, the very foundation of most current turbulence simulations. A Gibbsian statistical mechanics approach was developed for vortex motion by L. Onsager (1949) and for Fourier-truncated Euler equations by T.D. Lee (1951) and R.H. Kraichnan (1955). Despite important qualitative insights, arguably no quantitative statistical mechanics of turbulence has matured.

In this talk, a statistical physics closure[1] is proposed for Galerkin models of incompressible shear flows.[2] This closure employs a maximum entropy (MaxEnt) principle to infer the probability distribution in Galerkin state space using exact statistical balance equations as side constraints. We show how this closure generalizes Jaynes' (1957) derivation of the Maxwell-Boltzmann velocity distribution of molecules in a gas starting from Newton's laws. Application to an empirical Galerkin model of the periodic cylinder wake predicts mean amplitude values and modal energy levels in good agreement with direct numerical simulation. Recipes for more complicated Galerkin systems are provided.

The maximum-entropy closure has immediate consequences for reduced-order modelling, for fully nonlinear infinite horizon control of fluid flows, and, potentially, for turbulence theory. The talk comprises joint work with Robert Niven and Michael Schlegel.

[1] Noack, B. R. & Niven, R. 2011 Maximum-entropy closure for a Galerkin system of incompressible shear flow. Submitted to the *Journal of Fluid Mechanics*.

[2] Noack, B. R., Morzyński, M., & Tadmor, G. (editors) 2011 *Reduced-Order Modelling for Flow Control*. CISM Courses and Lectures n. **528**, Springer-Verlag Berlin.