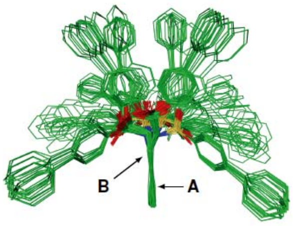
Visualizing Quantified Uncertainty



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Institut P', CNRS - Université de Poitiers - ENSMA, Poitiers, September 21, 2011

Outline

- Elementary Motivation for Considering Uncertainties
- The Bigger Picture
- Feature Extraction in Uncertain Data
- Iso-Contours in Uncertain Data
- Roads to Sharp Formulations, Future of Uncertainty Visualization

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Elementary Motivation

Numbers are Uncertain

All floating point numbers are afflicted with uncertainties

➔ 'true values' are not known exactly

- Measurements — values are uncertain
  due to non-perfect instruments, intrinsic randomness, …
- Simulation — results are uncertain
  due to uncertain parameters, inaccurate models, inaccurate numerics, rounding errors

Strictly spoken:

> Numbers
> without indication of **error bars or even PDF**
> are worthless !

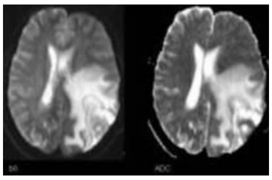
(fortunately: error bounds are often implicitly available by foreknowledge)

Example: Uncertainties in Biomedicine

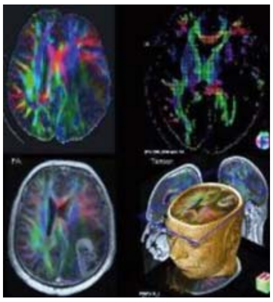
- Physical view
  - Quantitative science
  - No numbers without errors; some variables are random / considered as …
  - Uncertainties need to be specified
- Biological view
  - Huge variability of biological systems
  - Large variability of biological states
  - Regulation: intervals instead numbers
  - Many relevant variables are inaccessible
- Technical view
  - Measurements are riddled with error

Imagine, you are a Surgeon ...

- Patient: a human with a severe problem; his/her future well-being depends on you
- You have a highly responsible task
- You have to infer information from images, take decisions and perform actions
- You have to **rely** on
  - data acquisition
  - data processing
  - data visualization



Glioblastoma of the brain.
Displacement of nerves by the tumor.



Uncertainty in Visualization (More General)

All data with continuous range are affected by uncertainty.

- Conclusions have to be drawn from uncertain information
- This is the rule, not an exception
- Error estimation should be ubiquitous

➔ Visualization tools should show
- which information is reliable
- which information is uncertain

Data Analysis Point of View

In data analysis we tend

- to reconstruct fine details at the resolution limit and at marginal SNRs
- to extract complex features at the information limit
- to depict untrustworthy information…

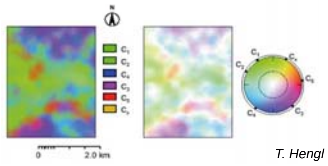
Visualization methods should

- sense these limits
- discern which information is reliable / which not
- convey visually the degree of trustworthiness of information

Previous Work (Few Samples ONLY)

- Statistical graphics: well developed

- Few 2D and 3D visualization methods exist



T. Hengl

- However: most 2D and 3D visualization methods still do **not** consider uncertainty

A. Pang, C. Wittenbrink,S. Lodha

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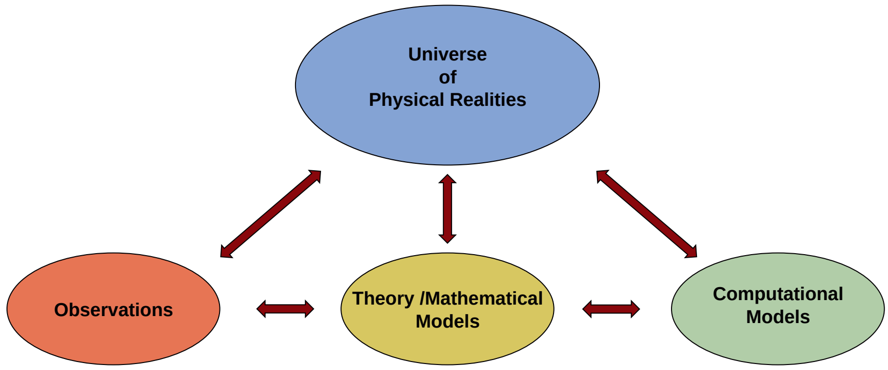
The Bigger Picture

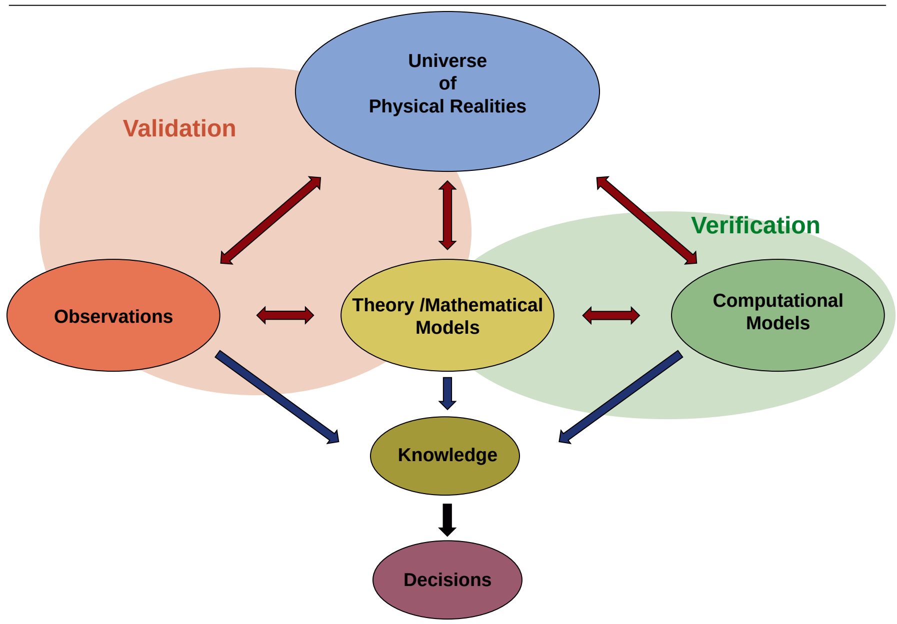
Three Pillars of Science ➔ Models



We form mental images of external objects and the form we give to them is such that the necessary consequences of the images in thought are always images of the necessary consequences in nature of the things pictured.

H. Hertz, in: Principles of Mechanics (1894)

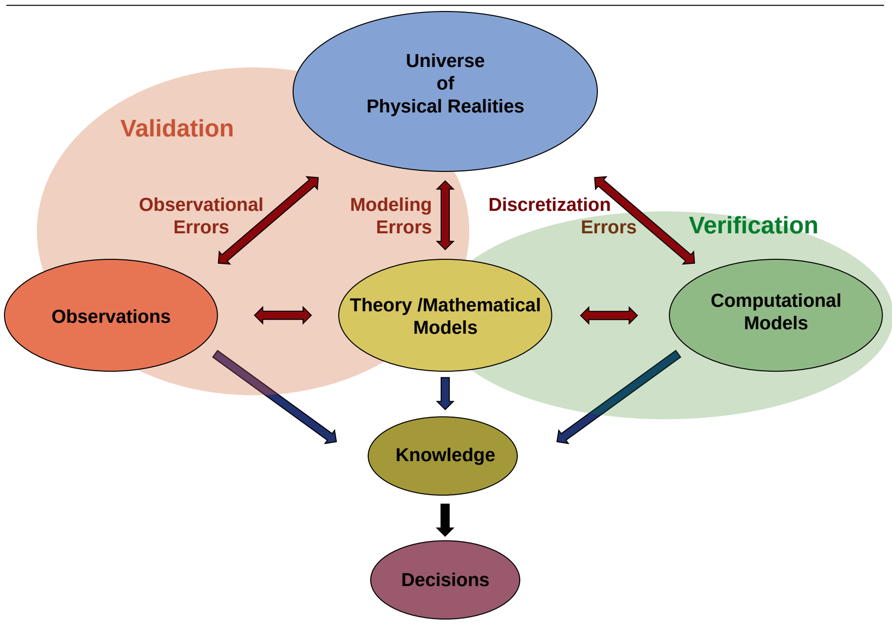
The Science Process: Validation & Verification



Creation of Theory, Knowledge

- David Hume (1711 – 1776)
  - Problem of induction: theories can not be validated
- Karl Popper (1902 – 1994)
  - Scientific theory can only be invalidated by contrary experimental evidence
  - Experimental observations are intrinsically interwoven into the scientific method
  - Scientific research is hypothesis-driven
  - Principle of falsification: only such hypotheses are legitimate that could be refuted by experimental evidence

The Science Process: Cascade of Uncertainties



Creation of Theory, Knowledge

Thomas Bayes' (1702–1761) ideas re-emerge:

- Bayesian analysis of computational predictions: uncertainty arises from
  - Model parameters m
    - Begin with priori joint PDF $\rho_M(m)$, describing what we know at the beginning
    - Calibrate via Bayesian inference: update the PDF to make the theory (model) consistent with particular observations (data)
  - Experimental observations d
    - Observational data d also will have uncertainties; represent by PDF $\rho_D(d)$
  - Theoretical model θ
    - Consider theory θ as conditional probability distribution $\theta(d \mid m)$ (‚likelihood') that maps the parameters m to a probability distribution of the outputs d

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Creation of Theory, Knowledge

- Bayes's theorem: Characterizes what is known about the model parameters expressed as the posterior PDF $\sigma(m \mid d)$ of the parameters m conditioned on the data d

$$\sigma(m \mid d) = \frac{\rho_M(m)\ \ \theta(d \mid m)}{\rho_D(d)}$$

- Statistically infers the posterior distribution $\sigma(\bullet \mid d)$ of parameters m that fit the theoretical model to the observations d
- Key to validation and – ultimately – to prediction with quantifiable uncertainties.

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Feature Extraction in Uncertain Data

Data Visualization

Two fundamental cases:

- Visualization of »raw« data
- Visualization of »features«

Visualizing Uncertain »Features«

Features = entities

- that can be computed from the input data
- that characterize certain aspects of the input data

Example: features in fields

	Spatial Features	Non-Spatial Features
Scalar	iso-surfaces	min/max; histogram
	ridges	entropy
	MS complex	
Vector	critical points	histogram on S^2xR
	integral curves	
	topological skeleton	
Tensor	eigenvector field lines	tensor DF

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Visualizing Uncertain »Features«

- Feature-based visualization
  - Definition of feature
  - Preprocessing of raw data: reconstruction, regularization → error propagation
  - Extraction of features → error propagation
  - Display of features → error propagation
- How to deal with uncertain features ?

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Example: Uncertain Iso-Surfaces

Questions:

- How do the errors propagate ?
  What is the resulting spatial uncertainty ?
  - Assume a sample grid.
  - What is the probability of a level crossing in a given cell element ? (edge, face, voxel, …)

- How to depict the spatial uncertainty ?

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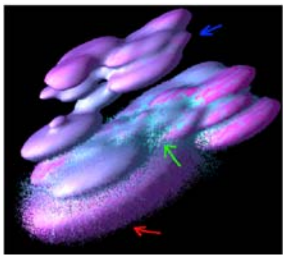
Iso-Contours in Uncertain Data

Positional Uncertainty of Iso-Contours: Condition Analysis and Probabilistic Measures
Kai Pöthkow, Hans-Christian Hege
IEEE Trans Comp Graph Vis, 17:10, Oct 2011, pp. 1393-1406

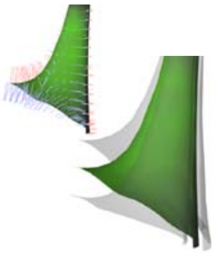
Probabilistic Marching Cubes
Kai Pöthkow, Britta Weber, Hans-Christian Hege
Computer Graphics Forum, 30:3, May 2011, pp. 931-940

Iso-Contours in Uncertain Data: Previous Work

Previous Work



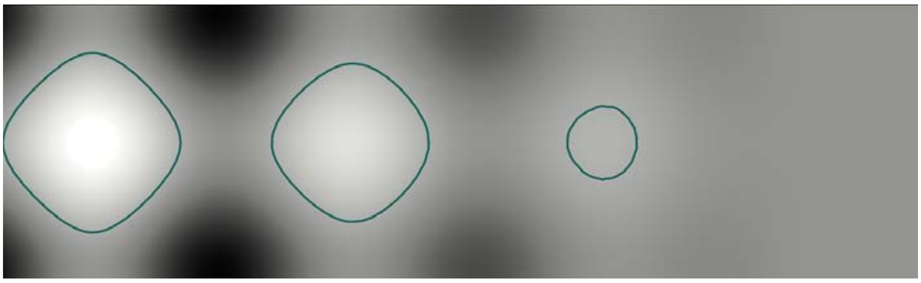
Grigoryan & Rheingans:
Probabilistic Surfaces: Point Based Primitives to Show Surface Uncertainty,
IEEE Trans Vis Comput Graph 10(5), pp. 564-573, 2004



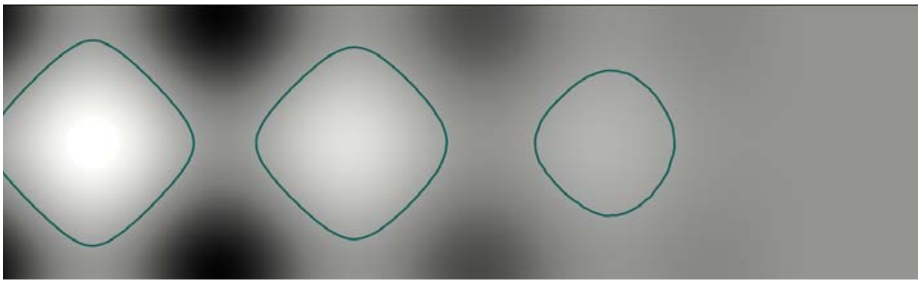
Zehner, Watanabe & Kolditz:
Visualization of Gridded Scalar Data with Uncertainty in Geosciences,
Comp. & Geosci 36(10), pp.1268-1275, 2010

Iso-Contours in Uncertain Data: Questions

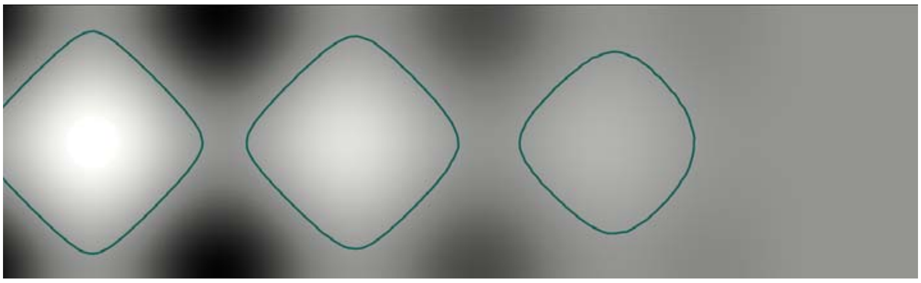
Iso-Surfaces in Noisy Data



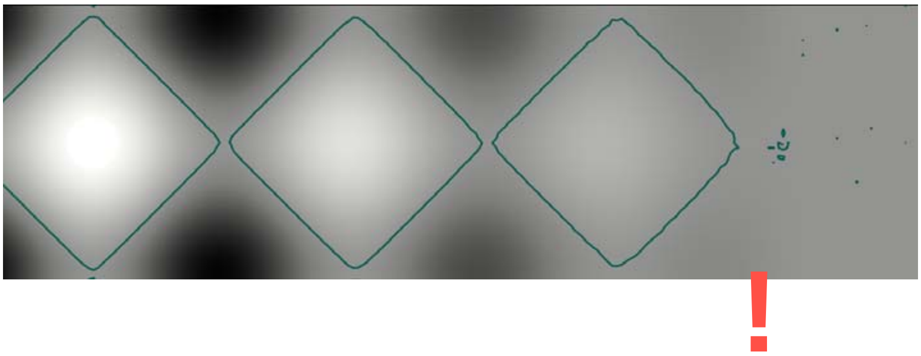
Iso-Surfaces in Noisy Data



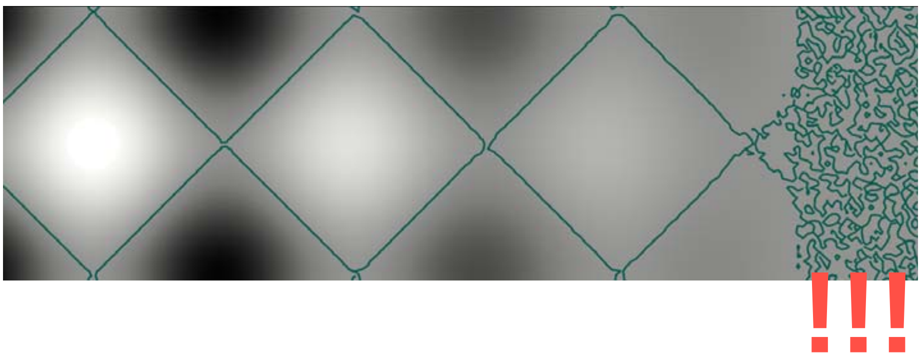
Iso-Surfaces in Noisy Data



Iso-Surfaces in Noisy Data



Iso-Surfaces in Noisy Data



Questions

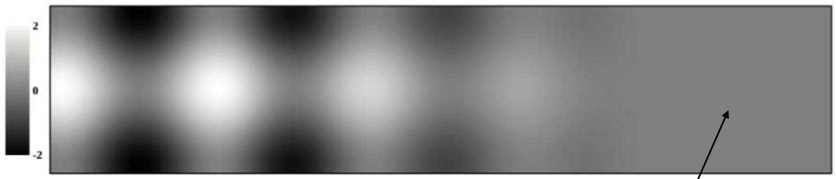
- How can we adequately model errors and uncertainty in scalar fields?
- What is the resulting spatial uncertainty of iso-contours?

Questions

- How can we adequately model errors and uncertainty in scalar fields?
- What is the resulting spatial uncertainty of isocontours?
- What is the impact of *spatial* correlation?

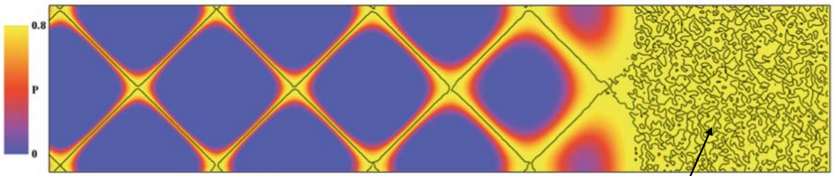
Solution we aim for …

Input



$g(x) + \epsilon(x)$ $\approx \vartheta$

Output



black curves: iso-contours $(g + \epsilon)^{-1}(\vartheta)$ ill defined

better: probability of iso-contour (color)

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Iso-Contours in Uncertain Data: Sensitivity Analysis

Sensitivity of Results to Changes in the Data

Problem: from x compute ρ

Condition number κ_{abs} : gives a measure of how sensitive the solution of a problem is to perturbations in the input data.

κ_{abs} is the smallest number with

perturbation

$$\kappa_{abs} \geq \frac{\|\rho(x)-\rho(x+\epsilon)\|}{|\epsilon|}$$

Common simplification: linearization. If ρ is differentiable then

$$\kappa_{abs} = \|\rho'(x)\|$$

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Error Amplification

How do iso-contours change if input data (scalar field g) is changed ?

Condition number of iso-surface computation $g^{-1}(\vartheta) = x$:

$$\kappa_{abs} = \frac{1}{\|\nabla g(x)\|}$$

Condition number can **diverge** !

- κ_{abs} is the amplification factor for absolute errors
- No surprise: only for regular values is $g^{-1}(\vartheta)$ a smooth d-1 manifold
- For iso-contour algorithms g needs to be a Morse function
- But even if g is a Morse function: problems occur for $\|\nabla g\| \gtrsim 0$

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Sensitivity of a Iso-Surface to Changes in the Data

Density of fuel during an injection process (data from www.volvis.org)

Iso-surface   Iso-surface, color: condition number

➔ If errors in the input data are unknown:

Condition number can give a hint where extracted feature might be uncertain due amplification of errors in the input data

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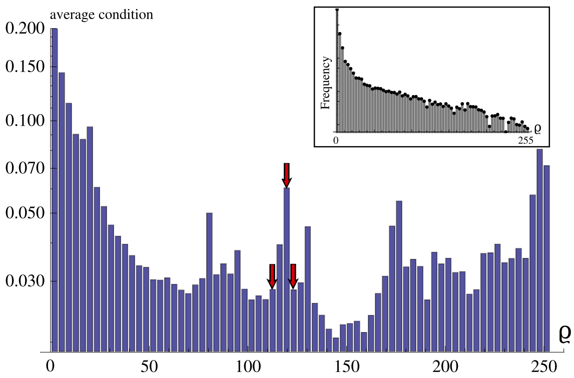
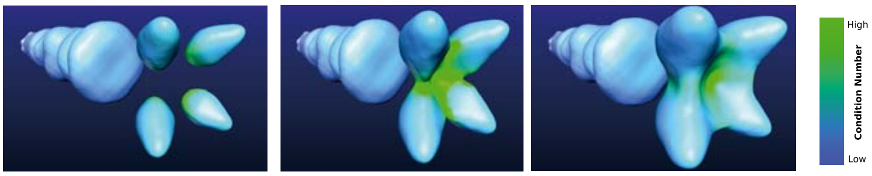
Selection of Well-Conditioned Iso-Surfaces





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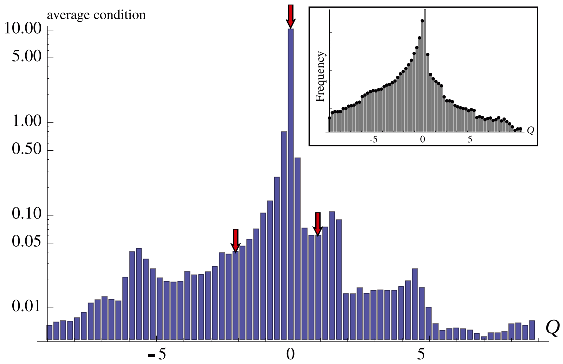
Selection of Well-Conditioned Iso-Surfaces

- Simulated flow around an airfoil (single timestep)
- Q field (Okubo-Weiss parameter)
  Q≈0 separates regions of dominant strain and vorticity
- Average condition numbers of iso-surfaces



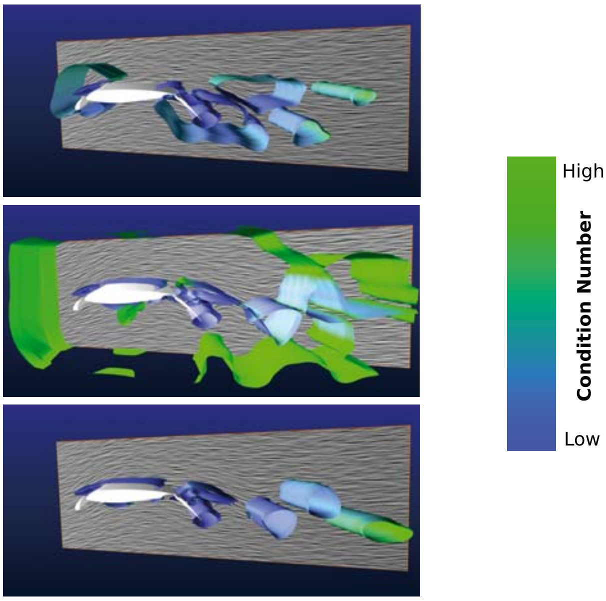
➔ Q≈0 most ill-conditioned iso-surface

Selection of Well-Conditioned Iso-Surfaces

$\theta = -2$

$\theta = 0$

$\theta = 1$



Iso-Contours in Uncertain Data: Random Field Model

Uncertain Iso-contours

- Scalar field $g: M \to \mathbb{R}$ on compact domain $M \subset \mathbb{R}^d$
- Consider level sets $\{x \in M \mid x = g^{-1}(\vartheta)\}$

- Discretely sampled on nodes $\{\mathbf{x}\}_{i \in I}$ with values $\{Y\}_{i \in I}$
- Let $\{Y\}_{i \in I}$ be random variables

  distributions f_i

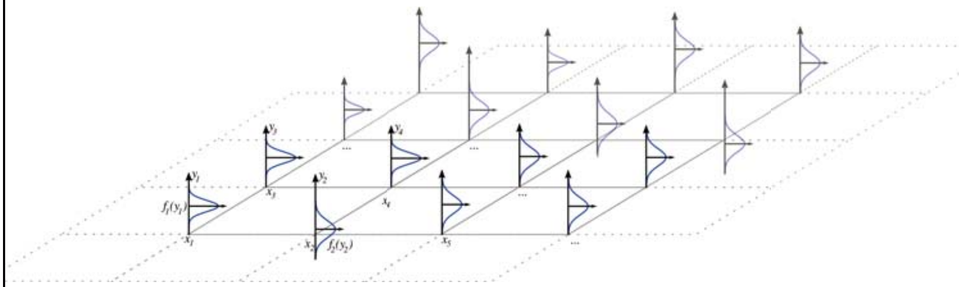
  means $\mu_i = \mathrm{E}(Y_i)$

  variances $\sigma_i^2 = \mathrm{E}(Y_i - \mu_i)^2$

  co-variances $\mathrm{Cov}(Y_i, Y_j) = \mathrm{E}((Y_i - \mu_i)(Y_j - \mu_j))$
- True values of field $g(\mathbf{x})$ unknown; assume: $g(\mathbf{x}_i) = \mu_i$

Gaussian Random Field

Discrete random field = multivariate Gaussian RV



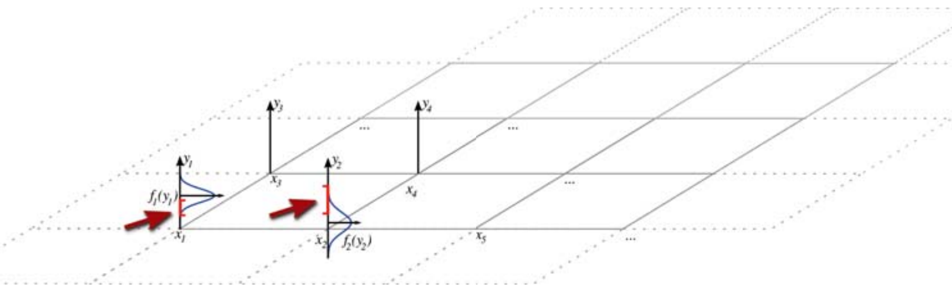
Gaussian Random Field

Discrete random field = multivariate Gaussian RV

$$\mathbf{Y} \sim \mathcal{N}_n(\mu, \Sigma) \qquad \begin{aligned} \mu &= [\mathrm{E}(Y_1), \mathrm{E}(Y_2), \ldots, \mathrm{E}(Y_n)] \\ \Sigma &= [\mathrm{Cov}(Y_i, Y_j)]_{i=1,2,\ldots,n;j=1,2,\ldots,n}. \end{aligned}$$

$$\mathbf{Y}(\mathbf{y}) = \frac{1}{(2\pi)^{n/2}\det(\Sigma)^{1/2}} \exp\left(-\tfrac{1}{2}(\mathbf{y}-\mu)^T\Sigma^{-1}(\mathbf{y}-\mu)\right)$$

Gaussian Random Field



Gaussian Random Field

Marginalization:

$$\int_{-\infty}^{\infty} \mathrm{d}y_{m+1} \cdots \int_{-\infty}^{\infty} \mathrm{d}y_n \, \frac{1}{(2\pi)^{n/2} \det(\Sigma)^{1/2}} \exp\left(-\tfrac{1}{2}(\mathbf{y}-\mu)^T \Sigma^{-1} (\mathbf{y}-\mu)\right)$$

$$= \frac{1}{(2\pi)^{m/2} \det(\tilde{\Sigma})^{1/2}} \exp\left(-\tfrac{1}{2}(\tilde{\mathbf{y}}-\tilde{\mu})^T \tilde{\Sigma}^{-1} (\tilde{\mathbf{y}}-\tilde{\mu})\right)$$

$$=: f_{\tilde{\mathbf{Y}}}(y_1, \ldots, y_m)$$

where $\tilde{\mathbf{Y}}$ is the reduced random vector and $\tilde{\mathbf{y}}, \tilde{\mu}$ and $\tilde{\Sigma}$
are the quantities $\mathbf{y}, \mu$ and Σ
with $n - m$ columns/rows deleted
that correspond to the marginalized variables $y_{m+1} \cdots y_n$

Gaussian Random Field

Complete random field

$$Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_m \\ \vdots \\ Y_n \end{bmatrix} \qquad \mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_m \\ \vdots \\ \mu_n \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \mathrm{Cov}_{1,1} & \cdots & \mathrm{Cov}_{1,m} & \cdots & \mathrm{Cov}_{1,n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathrm{Cov}_{m,1} & \cdots & \mathrm{Cov}_{m,m} & \cdots & \mathrm{Cov}_{m,n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathrm{Cov}_{n,1} & \cdots & \mathrm{Cov}_{n,m} & \cdots & \mathrm{Cov}_{n,n} \end{bmatrix}$$

Local marginal distribution

$$\tilde{Y} = \begin{bmatrix} Y_1 \\ \vdots \\ Y_m \end{bmatrix} \qquad \tilde{\mu} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_m \end{bmatrix} \qquad \tilde{\Sigma} = \begin{bmatrix} \mathrm{Cov}_{1,1} & \cdots & \mathrm{Cov}_{1,m} \\ \vdots & \ddots & \vdots \\ \mathrm{Cov}_{m,1} & \cdots & \mathrm{Cov}_{m,m} \end{bmatrix}$$

Probabilities of Classes of Realizations

Constrain $m \leq n$ RV Y_i to subsets S_i .

Re-order RV such that constrained ones are the first m ones.

Probability of constrained realization:

$$\mathrm{Prob}\,(Y_1 \in S_1, \ldots, Y_m \in S_m) =$$

$$\int_{S_1} \mathrm{d}y_1 \ldots \int_{S_m} \mathrm{d}y_m \int_{\mathbb{R}} \mathrm{d}y_{m+1} \ldots \int_{\mathbb{R}} \mathrm{d}y_n \; f_{\mathbf{Y}}(y_1, \ldots, y_n)$$

For Gaussian distribution:

$$\int_{S_1} \mathrm{d}y_1 \ldots \int_{S_m} \mathrm{d}y_m \; f_{\tilde{\mathbf{Y}}}(y_1, \ldots, y_m)$$

Iso-Contours in Uncertain Data: Level-Crossing Probabilities

Level Crossing Probabilities

Assume C^0 interpolant g for any realization (= grid function) which takes its extreme values at the sample points.

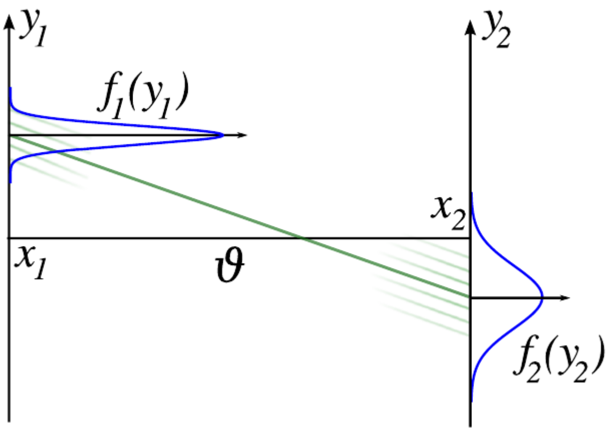
Consider grid cell c with indices $\tilde{I} \in I$.

Cell c crosses ϑ-level of $g_{\{y\}}$ if and only if not all differences $(y_i - \vartheta)_{i \in \tilde{I}}$ have the same sign.

Level crossing probability $\mathrm{Prob}_c(\vartheta\text{-crossing})$:
Integrate $\{Y\}_{i \in \tilde{I}}$ over sets $\{y_j \in \mathbb{R} \,|\, y_j \geq \vartheta\}$ and $\{y_i \in \mathbb{R} \,|\, y_i \leq \vartheta\}$

Alternatively:

Level Crossing Probabilities on **Edges**



Level Crossing Probabilities on **Edges**

Edge with bivariate Gaussian RV $\mathbf{Y} = [Y_1, Y_2]$

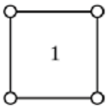
$$\mathrm{Prob}_c(\vartheta\text{-crossing}) =$$

$$= \mathrm{Prob}(Y_1 \leq \vartheta, Y_2 > \vartheta) \quad + \ \mathrm{Prob}(Y_1 > \vartheta, Y_2 \leq \vartheta)$$

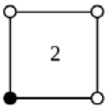
$$= \int\limits_{y_1 \leq \vartheta} \mathrm{d}y_1 \int\limits_{y_2 > \vartheta} \mathrm{d}y_2 \ f_{\mathbf{Y}}(y_1, y_2) + \int\limits_{y_1 > \vartheta} \mathrm{d}y_1 \int\limits_{y_2 \leq \vartheta} \mathrm{d}y_2 \ f_{\mathbf{Y}}(y_1, y_2)$$

Level Crossing Probabilities on **Faces**

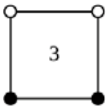
4 Cases (after Symmetry Reduction) $\triangleq$ **Corresponding Integrals**



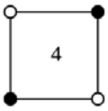
$$P_{\vartheta,1} = \underset{(y_1 > \vartheta \wedge y_2 > \vartheta \wedge y_3 > \vartheta \wedge y_4 > \vartheta)}{\int dy_1 \int dy_2 \int dy_3 \int dy_4} \; f_{\mathbf{Y}}(y_1, y_2, y_3, y_4)$$



$$P_{\vartheta,2} = \underset{(y_1 \leq \vartheta \wedge y_2 > \vartheta \wedge y_3 > \vartheta \wedge y_4 > \vartheta)}{\int dy_1 \int dy_2 \int dy_3 \int dy_4} \; f_{\mathbf{Y}}(y_1, y_2, y_3, y_4)$$



$$P_{\vartheta,3} = \underset{(y_1 \leq \vartheta \wedge y_2 \leq \vartheta \wedge y_3 > \vartheta \wedge y_4 > \vartheta)}{\int dy_1 \int dy_2 \int dy_3 \int dy_4} \; f_{\mathbf{Y}}(y_1, y_2, y_3, y_4)$$

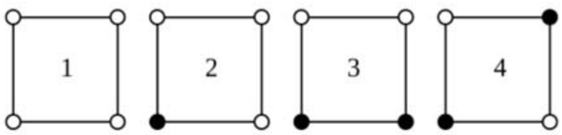


$$P_{\vartheta,4} = \underset{(y_1 \leq \vartheta \wedge y_2 > \vartheta \wedge y_3 \leq \vartheta \wedge y_4 > \vartheta)}{\int dy_1 \int dy_2 \int dy_3 \int dy_4} \; f_{\mathbf{Y}}(y_1, y_2, y_3, y_4)$$

Level Crossing Probabilities on Rectangular **Cells**, …

Types of integrals $\triangleq$ symmetry-reduced Marching cubes cases.

In 2D: 4 distinct cases (1 non-crossing, 3 crossing)



In 3D: 15 distinct cases (1 non-crossing, 14 crossing)

In 4D: 223 distinct cases (1 non-crossing, 222 crossing)

In nD: use Polya's counting theory

Level Crossing Probabilities – Simplified

of cases (i.e. integrals) **with** level crossings grows with dimension …

Better exploit $\mathrm{Prob}_c(\vartheta\text{-crossing}) = 1 - \mathrm{Prob}_c(\vartheta\text{-non-crossing})$

↑

only **2** cases **without** level crossings
➜ only **2** integrals for all dimensions !

e.g. for square cells in 2D:

$$\mathrm{Prob}_c(\vartheta\text{-crossing}) = 1 - \int_{\substack{(y_1 \leq \vartheta \wedge y_2 \leq \vartheta \wedge y_3 \leq \vartheta \wedge y_4 \leq \vartheta) \\ \vee (y_1 > \vartheta \wedge y_2 > \vartheta \wedge y_3 > \vartheta \wedge y_4 > \vartheta)}} \mathrm{d}y_1 \int \mathrm{d}y_2 \int \mathrm{d}y_3 \int \mathrm{d}y_4 \; f_{\mathbf{Y}}(y_1, y_2, y_3, y_4)$$

But dimension of integrals still = # vertices of geometric object !

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Iso-Contours in Uncertain Data: Algorithm & Implementation

Algorithm & Implementation

- Preprocessing
  - Estimate $\hat{\mu}_i$ for all sample points
  - Estimate $\widehat{\mathrm{Cov}}_{i,j}$ for all 2- or 3-cells
- For a given iso-value ϑ
  - Estimate crossing probabilities using Monte Carlo integration

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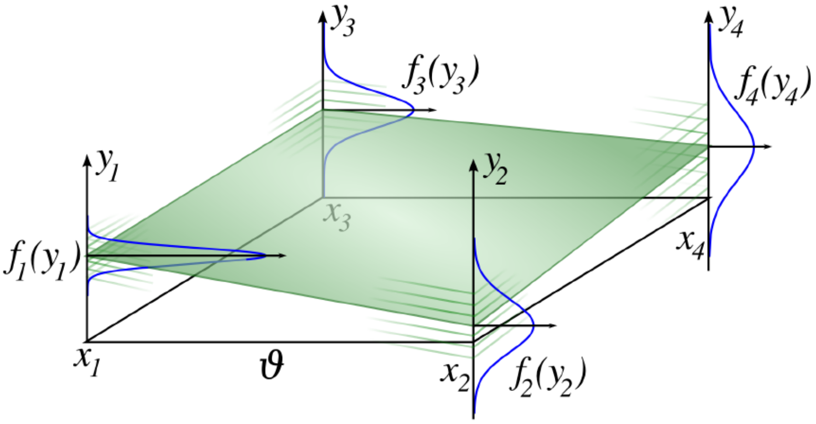
Level Crossing Probabilities on Faces



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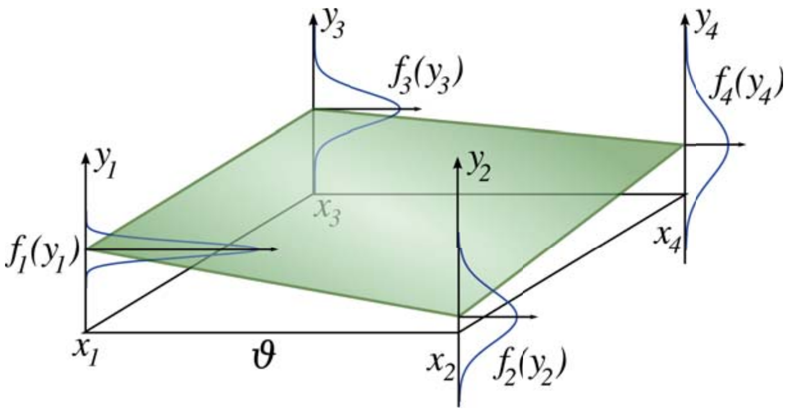
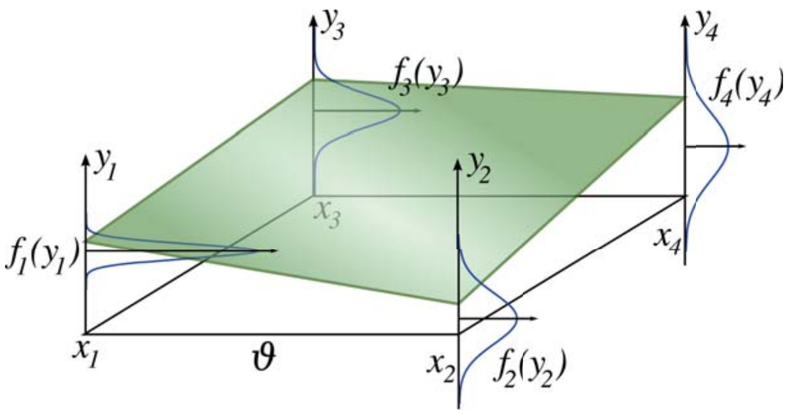
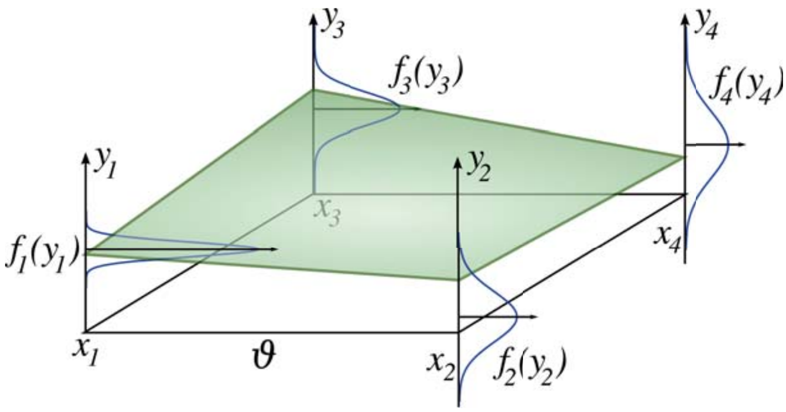
Monte Carlo Step



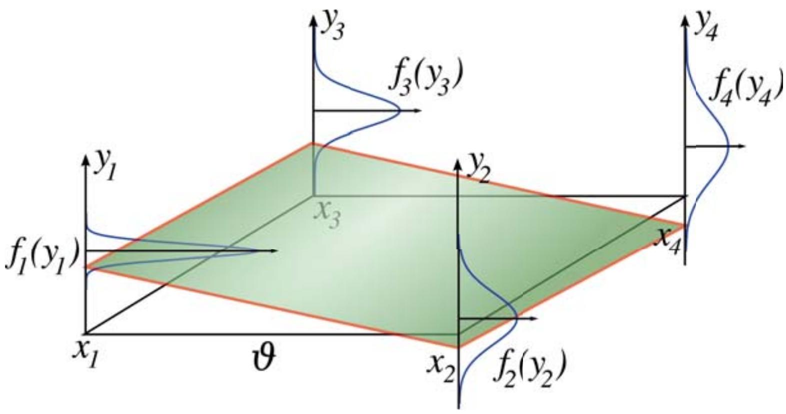
Monte Carlo Step



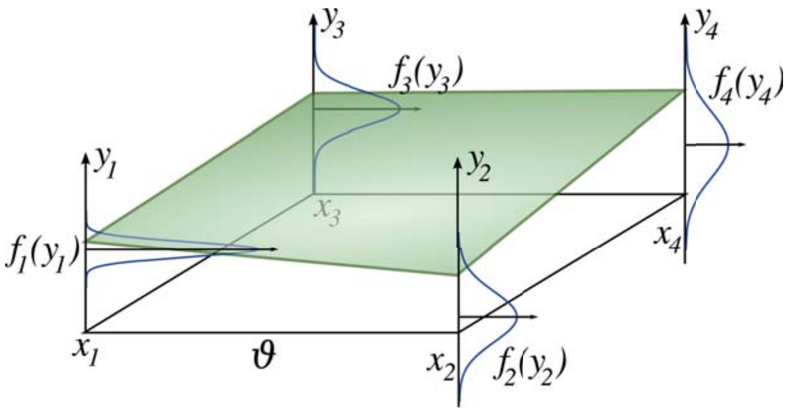
Monte Carlo Step



Monte Carlo Step



Monte Carlo Step



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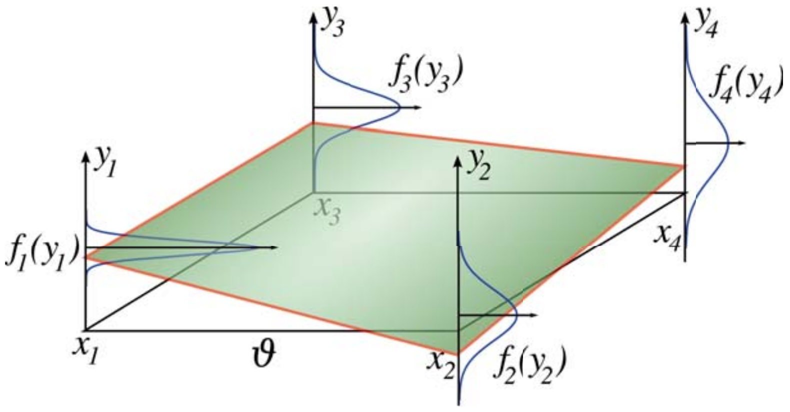
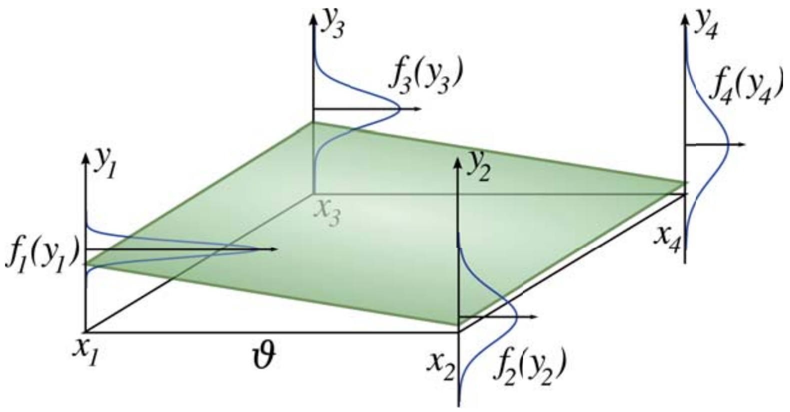
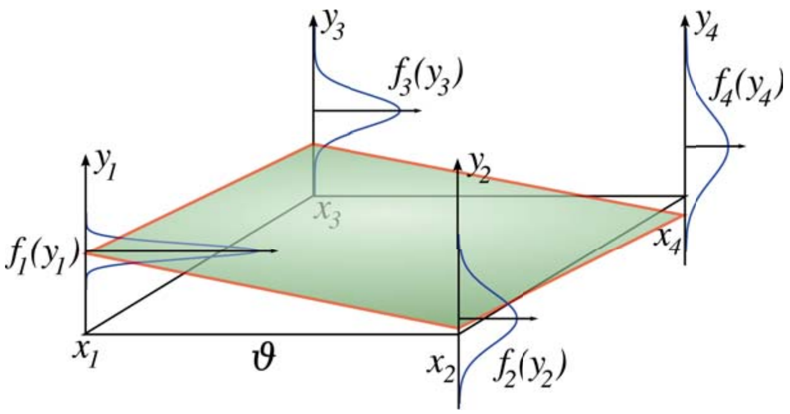


Monte Carlo Step



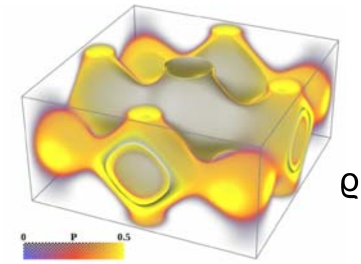
Monte Carlo Step



$$
\begin{array}{l}
\text{for each cell } c \ \{ \\
\quad L_c \leftarrow \text{CholeskyDecomposition}(\Sigma_c) \\
\quad \#\text{crossings} \leftarrow 0 \\
\quad \text{for } 1 \ldots \#\text{samples} \ \{ \\
\qquad \mathbf{y} \leftarrow \text{random numbers } y_1 \ldots y_m \sim \mathcal{U}(0,1) \\
\qquad \mathbf{y} \leftarrow \text{BoxMullerTransform}(\mathbf{y}) \\
\qquad \mathbf{y} \leftarrow L_c \, \mathbf{y} + \mu_c \\
\qquad \text{if}(\text{crossing}_\vartheta(\mathbf{y})) \ \#\text{crossings} \leftarrow \#\text{crossings} + 1 \\
\quad \} \\
\quad \text{Prob}_c \leftarrow \#\text{crossings}/\#\text{samples} \\
\}
\end{array}
$$

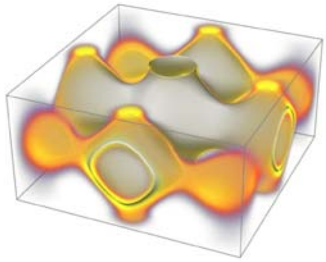
Iso-Contours in Uncertain Data: Results

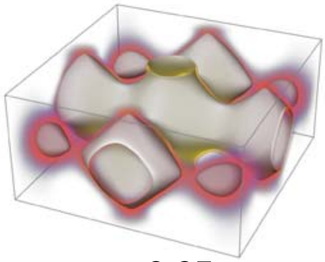
Impact of Spatial Correlations



synthetic data

ϱ=0.00

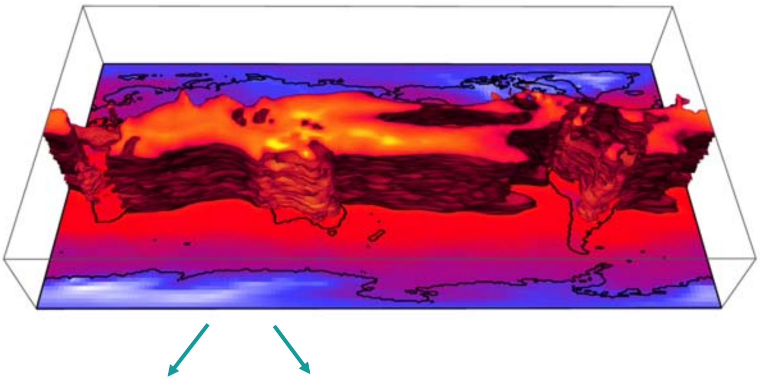




ϱ=0.65 ϱ=0.95

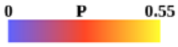
Climate Simulation

Data courtesy of ECMWF

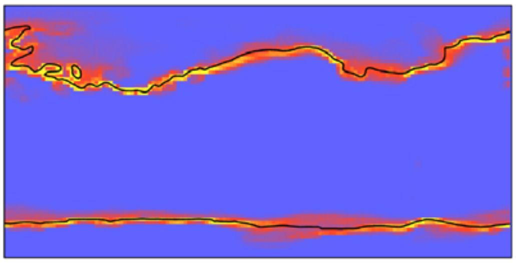


$\hat{\mu}_i$ $\widehat{\mathrm{Cov}}_{i,j}$

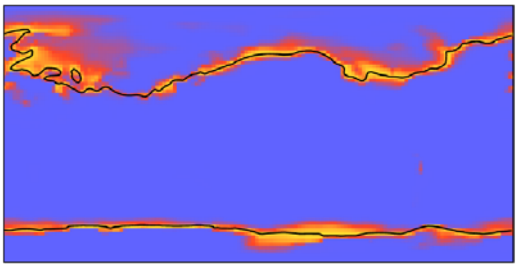
Isotherm of Climate Simulation



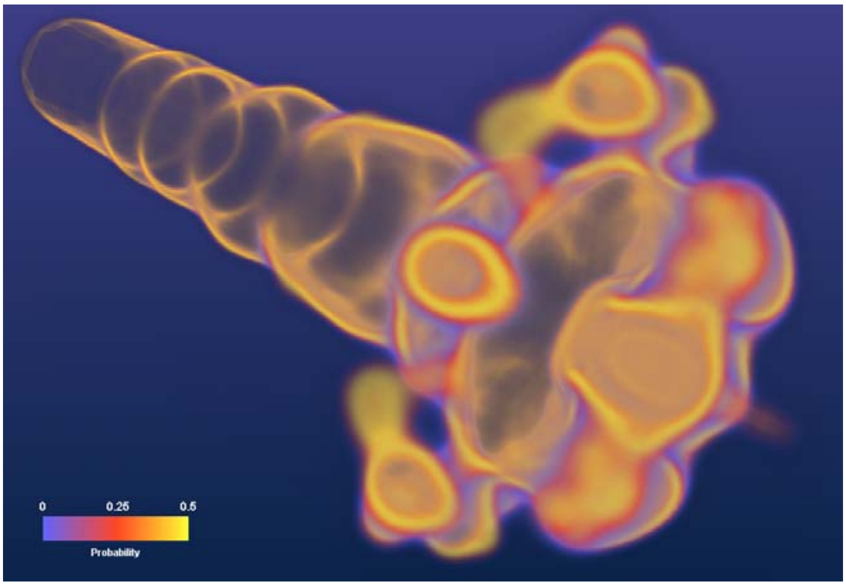
spatial correlations considered



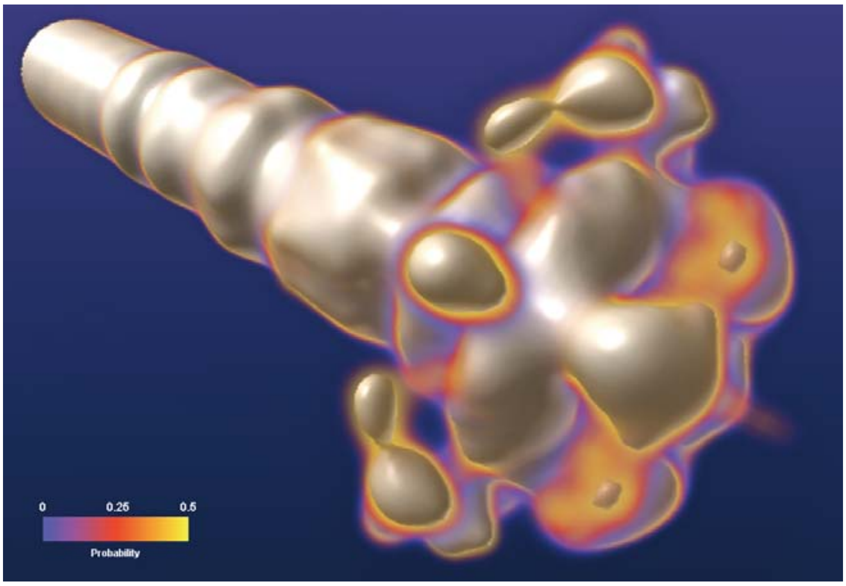
not considered



Fuel Injection Data Set + Artificial Noise: Uncertain Level Set



Fuel Injection Data Set + Artificial Noise: Uncertain Level Set



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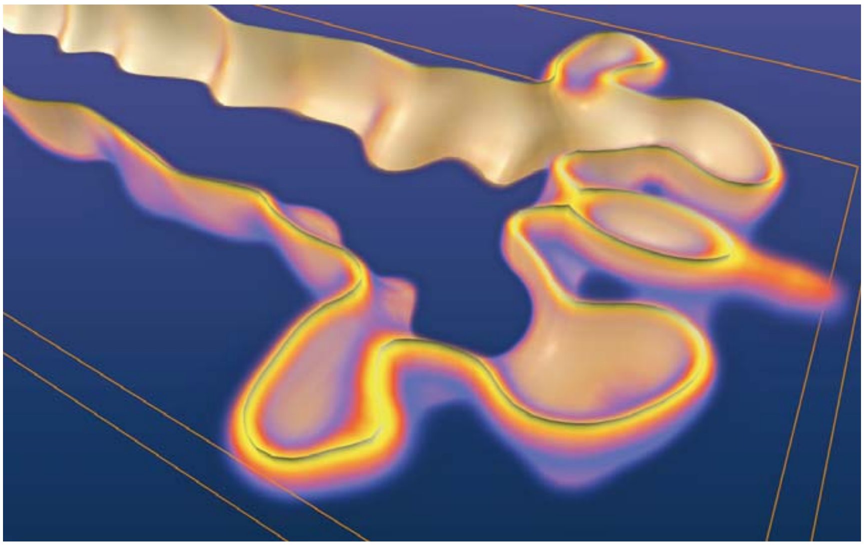
Fuel Injection Data Set + Artificial Noise: Uncertain Level Set



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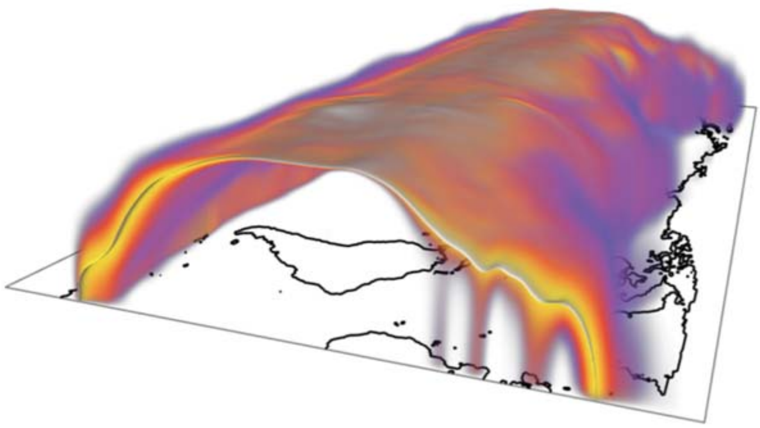
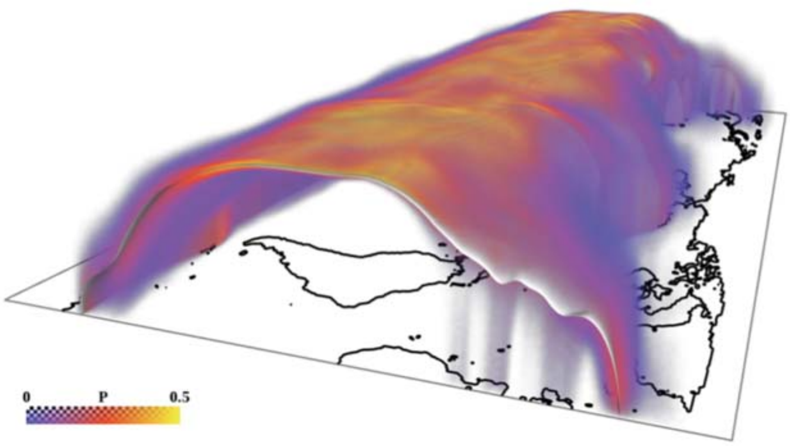
Application Example: Isotherm of Climate Simulation



spatial correlations not considered

Application Example: Isotherm of Climate Simulation



spatial correlations considered

Roads to Sharp Formulations − Future of Uncertainty Vis

We need to understand …

- Types of uncertainty
  - Aleatoric — experimenter: results differ, each time she/he runs an experiment; modeller: does not foresee the possibility to reduce uncertainty
  - Epistemic — experimenter: we could in principle know, but don't know in practice; modeller: sees the possibility to reduce uncertainty by gathering more data or be refining models
- Uncertainty representations
  - Intervals ➔ interval computing
  - Fuzzy numbers, sets ➔ soft computing
  - Probabilities, PDFs ➔ probability theory, statistics
- Reasoning under uncertainty, decision support
  - Formal reasoning ➔ statistical inference ➔ uncertainty in AI
  - Defuzzification, decision taking ➔ risk & decision theory

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We need to understand and further develop …

Uncertainty quantification in modelling and simulation

- Estimate parameter uncertainty
- Develop statistical / fuzzy models
- Analyse uncertainty propagation
- Perform sensitivity analysis and dimensional reduction
- Develop methods for defuzzification
- Develop tools to support in decision making

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We need to understand and further develop …

Uncertainty Visualization

- UQ in the visualization pipeline
- Fuzzy analogues of crisp features, uncertainty of features
- Visual mapping of uncertain data and fuzzy features
- Evaluation of uncertainty representations (perception, cognition)
- Visual support for data processing techniques:
  data aggregation, ensemble analysis, …
- Visual support of defuzzification
- Visual support in decision making
- Evaluation of uncertainty VIS / VA systems

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Conclusion

- Major tasks in uncertainty visualization
  - Uncertainty quantification in visualization pipeline
  - Visual mapping of uncertain data and fuzzy features
  - Support in decision making
- Uncertain features
  - Condition numbers, sensitivity analysis
  - Probabilistic formulation

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Conclusion

- Uncertain iso-surfaces
  - reveal information not visible before
- Assumption of certain distribution law
  - arbitrary number of realizations possible
  - more details than with limited number of realizations
- Advantage of not computing crisp iso-surfaces:
  - no regularity requirements (Morse, non-Morse)
  - no special cases in algorithm for degenerate cases
- Most important research questions:
  - visual mapping
  - non-Gaussian random fields

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Thank you very much for your attention !

www.zib.de/visual

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